SDMA Group Selection for Reduced Computational Complexity on MU MIMO Systems

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Abstract. *Optimizing wireless systems with Multiple Input Multiple Output (MIMO) by means of Radio Resource Allocation (RRA) in general requires very complex algorithms. In this work, we revisit two Radio Resource Allocation (RRA) problems in this scenario: Unconstrained Rate Maximization (URM) and Constrained Rate Maximization (CRM). In order to optimally solve those problems it is necessary to list all possible Space Division Multiple Access (SDMA) groups combinations and calculate the transmit and receive filters of the terminals for each combination. The main proposal in this work is to decrease the complexity of the RRA solutions at the cost of a controlled performance degradation by selecting only a fraction of all possible SDMA groups.*

1. Introduction

The advantages of Multiple Input Multiple Output (MIMO) technology over single antenna systems have become apparent to wireless research community mainly after the seminal works of Foschini [Foschini 1996] and Alamouti [Alamouti 1998] at the end of 1990's. Almost twenty five years later, MIMO technology have become mandatory in modern communication standards such as Long Term Evolution (LTE)-Advanced and Worldwide Interoperability for Microwave Access (WiMAX) [Li et al. 2010].

The MIMO technology together with Orthogonal Frequency Division Multiple Access (OFDMA) can be used to obtain gains over single antenna systems in transmit data rate, bit error rates and co-channel interference mitigation [Mietzner et al. 2009]. Multiuser (MU) MIMO schemes, also known as Space Division Multiple Access (SDMA), consist in the use of multiple antennas to enable the allocation of different spatial subchannels to different terminals in the same time-frequency resource [Gesbert et al. 2007]. In this case, the spatial dimension can be used as another tool to exploit the multiuser diversity.

The joint use of MIMO and Radio Resource Allocation (RRA) is a relevant strategy to deal with the challenges of next generation of cellular networks. However, the MU MIMO capability turns the RRA problems even more challenging due to the added degree of freedom and, therefore, requires computationally expensive solutions. In this work we revisit two RRA problems in MU MIMO scenario: Unconstrained Rate Maximization (URM) and Constrained Rate Maximization (CRM) [Lima et al. 2014]. An important problem in the literature is the total data rate (sum rate) maximization or URM. The achievable sum rate capacity for the multi-antenna downlink channel has been found in [Caire and Shamai 2003] while [Tejera et al. 2006] shows how that sum rate capacity can be found using a non-linear processing technique called Dirty Paper Coding (DPC) [Costa 1983]. However, obtaining the optimal transmission policy when employing DPC is a computationally complex non-convex problem. Motivated by this, linear processing at the transceivers has been adopted due to the good achieved performance-complexity trade off [Ho and Liang 2009]. The Block Diagonalization (BD)-Zero Forcing (ZF) strategy is a linear filtering scheme capable of spatially multiplexing multiple terminals in the same frequency resource in a MU-MIMO [Spencer et al. 2004]. As will be shown later, in order to obtain the optimal solution of the URM problem when linear spatial filtering is used, we need to calculate the transmit and receive filters of the terminals of all possible SDMA group combinations in all available frequency resources.

Although URM solution achieves the maximum spectral efficiency, it is unfair which leads to service starvation to terminals with unfavorable channel conditions. The total data rate maximization subject to satisfaction constraints or CRM has been considered in [Lima et al. 2014]. The objective of CRM is the same as the URM problem, however, the former assumes a multiservice scenario where each terminal is assumed to be transmitting or receiving data from a multimedia service, e.g., Voice over IP (VoIP) or web browsing. Therefore, besides maximizing the total transmit data rate, the resource assignment should guarantee that a minimum number of terminals of each service is satisfied with the provided Quality of Service (QoS). In order to obtain the optimal solution of CRM problem, the authors have shown that the original non-linear optimization problem can be converted to an Integer Linear Program (ILP) and then solved by Branch-and-Bound (BB)-based algorithms [Nemhauser and Wosley 1999]. However, as in the URM problem, the linear spatial filters at the transmitter and receiver of all terminals in all possible SDMA groups and frequency resources should be pre-calculated. This operation is computationally intense for both URM and CRM problems.

The main contribution of this article is the proposal of methods to pre-select a fraction of all possible SDMA groups based on simple and reasonable metrics reducing the problem size proposed in [Lima et al. 2014]. The selected SDMA groups will be used in the solutions of the URM and CRM problems. With this approach, only the spatial filters for the terminals of the selected SDMA groups should be calculated. Besides saving the computational task of calculating filters of terminals in some SDMA groups, the solutions of the URM and CRM problems will have their computational complexity decreased. The main contributions of this work are: Proposal of a framework to reduce the computational complexity to obtain the optimal solutions of problems URM and CRM; Proposal of metrics to measure the relevance of SDMA groups; Performance evaluation of the proposed framework and grouping metrics; Calculation of the computational complexity of the solutions to URM and CRM problems with our proposed framework.

The remainder of this article is organized as follows. In section 2, we provide the system model and main assumptions. The proposed solution to decrease the computational complexity is presented in section 3 and performance results based on simulations are shown in section 4. Finally, the main conclusions and perspectives are depicted in sections 5.

2. System Modeling

2.1. Definitions and variables

Most of the system scenario and variables are similar to the ones presented in [Lima et al. 2014]. Basically, we assume RRA in the downlink of a given cell of an OFDMA cellular system. The minimum allocable resource, or Resource Block (RB), consists in a group of one or more adjacent subcarriers and a number of consecutive Orthogonal Frequency Division Multiplexing (OFDM) symbols in the time domain, which represents the Transmission Time Interval (TTI). We assume a MU MIMO scenario with M_T and M_R antennas at the Base Station (BS) and terminals, respectively. There are J terminals in the considered cell and N available RBs. $\mathcal{J}$ and $\mathcal{N}$ are the set of available terminals and RBs, respectively. As the CRM problem assumes a multiservice scenario, we consider that each terminal is using a service $s \in \mathcal{S}$, e.g., web browsing or file download, where $\mathcal{S}$ is the set of all services. The set of terminals using service s is given by $\mathcal{J}_s$. The number of terminals from service s is $|\mathcal{J}_s| = J_s$.

The MIMO channel of the link between the BS and terminal j on RB n is modeled by the matrix $H_{j,n}$ that is composed of the elements $h_{j,n,a,b}$ that represents the channel transfer function between the a^{th} receive antenna of terminal j and the b^{th} transmit antenna of the BS on RB n. The maximum number of orthogonal spatial subchannels that can be used per RB in the considered cell is $\min\left(J\, M_R, M_T\right)$. The Figure 1 is a scenario representation in which there are two terminals with two receiver antennas each and 4 transmission antennas in the BS.

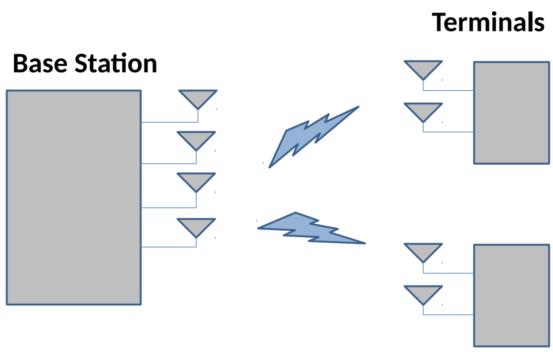

Figure 1. Transmission between a BS with 4 antennas and two terminals with 2 antennas.

We define SDMA group as a set of terminals spatially multiplexed in a given RB. Assuming, for example, three terminals and $M_T = M_R = 2$, the possible SDMA groups that can be built are $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$. In this sense, $\mathcal{G}$ consists in a set with the indices of all SDMA groups that can be built. The group $\{1,2,3\}$ cannot be considered because it has more terminals than antennas in the BS. In the previous example, the cardinality of $\mathcal{G}$ is $G = 6$. Assume that $\mathcal{G}_n$ is the set with the terminal's indices of the selected SDMA group assigned to RB n. The symbols to be transmitted to terminal $j \in \mathcal{G}_n$ are filtered by a transmit matrix $\mathbf{M}_{j,n}$ whereas at the receiver they are filtered by a receive matrix $\mathbf{D}_{j,n}$. Note that $c_{j,n}$ is the number of streams transmitted to terminal j on RB n where $c_{j,n} \leq \min(M_T, M_R, \nu_{j,n})$, whereas $\nu_{j,n}$ is the rank of the channel matrix $\mathbf{H}_{j,n}$. Therefore, the input-output relation for the MIMO channel is given by

$$\widehat{\mathbf{b}}_{j,n} = \mathbf{D}_{j,n}\widetilde{\mathbf{b}}_{j,n} = \alpha_j \mathbf{D}_{j,n}\mathbf{H}_{j,n}\mathbf{M}_{j,n}\mathbf{b}_{j,n} + \alpha_j \mathbf{D}_{j,n}\mathbf{H}_{j,n} \sum_{i \in \mathcal{G}_n, i \neq j} \left(\mathbf{M}_{i,n}\mathbf{b}_{i,n}\right) + \mathbf{D}_{j,n}\mathbf{n}_{j,n}, \quad (1)$$

where $\widetilde{\mathbf{b}}_{j,n}$ and $\widehat{\mathbf{b}}_{j,n}$ are the prior-filtering and the post-filtering received signal vector of terminal j on RB n, respectively, $\mathbf{b}_{j,n}$ is the transmit signal vector of terminal j on RB n, $\mathbf{n}_{j,n}$ is the white Zero Mean Circularly Symmetric Complex Gaussian (ZMCSCG) noise vector of terminal j on RB n, and α_j represents the joint effect of the path loss and shadowing.

The Signal to Interference plus Noise Ratio (SINR) of the l^{th} stream of terminal j on RB n, $\gamma_{j,n,l}$, is given by

$$\gamma_{j,n,l} = \frac{\|\sqrt{\alpha_j}\mathbf{d}^l_{j,n}\mathbf{H}_{j,n}\mathbf{M}_{j,n}\mathbf{b}_{j,n}\|_2^2}{\|\sqrt{\alpha_j}\mathbf{d}^l_{j,n}\mathbf{H}_{j,n}\sum_{i \in \mathcal{G}_n, i \neq j}\left(\mathbf{M}_{i,n}\mathbf{b}_{i,n}\right) + \mathbf{d}^l_{j,n}\mathbf{n}_{j,n}\|_2^2}, \quad (2)$$

where $\mathbf{d}^l_{j,n}$ is the l^{th} row of matrix $\mathbf{D}_{j,n}$ and $\|\cdot\|_2$ denotes the 2-norm of a vector. We assume that the transmit power is fixed and equally distributed among the spatial subchannels.

Consider that the BS employs a link adaptation functionality that selects the best Modulation and Coding Scheme (MCS). Note that the choice of the best MCS depends on the channel transfer function as well as on the prior knowledge of the transmit and receive filters. Assuming that the chosen SDMA group for RB n, $\mathcal{G}_n$, corresponds to the g^{th} SDMA group in $\mathcal{G}$, the total transmit data rate of terminal j that belongs to the $g^{\text{th}} \in \mathcal{G}$ SDMA group is given by

$$r_{g,j,n} = \sum_{\forall l \in \mathcal{L}_{j,n}} f\left(\gamma_{j,n,l}\right), \quad (3)$$

where $f(\cdot)$ represents the link adaptation function and $\mathcal{L}_{j,n}$ is the set of spatial subchannels of terminal j on RB n.

We define $\mathbf{X}$ as an assignment matrix with binary-valued elements $x_{g,n}$ that assume 1 if SDMA group $g \in \mathcal{G}$ is assigned to RB $n \in \mathcal{N}$, and 0 otherwise. Let $\mathbf{O}$ be a binary matrix with elements $o_{g,j}$ that assume 1 if terminal j belongs to SDMA group g, and 0 otherwise. We assume that at the current TTI, terminal j has a data rate requirement equal to t_j, that a minimum of k_s terminals should be satisfied for service s and that the indices of the terminals in $r_{g,j,n}$, $o_{g,j}$ and in t_j are sequentially disposed according to the service i.e., the flows from j = 1 to $j = J_1$ are from service 1, flows from $j = J_1 + 1$ to $j = J_1 + J_2$ are from service 2, and so on.

2.2. BD-ZF

BD-ZF is a spatial filtering scheme whose main idea is to provide transmit and receive filters that enable to simultaneously transmit to multiple terminals in the same frequency resource (MU MIMO) without multi-user interference [Spencer et al. 2004]. It is a generalization of ZF precoding scheme for multi-antenna terminals in which multi-user interference is eliminated by projecting the signal of each terminal onto the kernel of the joint null space of the channels of all other terminals sharing the same RB. For more details, the interested reader can see [Spencer et al. 2004, Lima et al. 2014].

The dominant operation in the BD-ZF algorithm is the Singular Value Decomposition (SVD) performed on each terminal's channel matrix. According to [Björck 1996], the worst-case computational complexity when the SVD decomposition is applied in a matrix with dimension $M_R \times M_T$ is $\mathcal{O}\left(2M_R M_T^2 + 4M_T^3\right)$. Assuming that the SDMA group has J' terminals, the SVD is applied on the J' channel matrices and, therefore, the worst-case computational complexity is $\mathcal{O}\left(J'2M_R M_T^2 + J'4M_T^3\right)$.

2.3. URM problem and solution

According to the definitions presented before the URM problem can be formulated as follows [Lima et al. 2014]:

$$\max_{\mathbf{X}} \left(\sum_{g \in \mathcal{G}} \sum_{n \in \mathcal{N}} \sum_{j \in \mathcal{J}} x_{g,n}\, o_{g,j}\, r_{g,j,n} \right), \tag{4a}$$

subject to

$$\sum_{g \in \mathcal{G}} x_{g,n} = 1, \quad \forall n \in \mathcal{N}, \tag{4b}$$

$$x_{g,n} \in \{0, 1\}, \quad \forall g \in \mathcal{G} \text{ and } \forall n \in \mathcal{N}. \tag{4c}$$

The objective function shown in (4a) is the total downlink data rate transmitted by the BS. The two constraints (4b) and (4c) assure that an RB will not be shared by different SDMA groups. The optimal solution of URM problem can be found according to the following steps:

1. Calculate $r_{g,j,n}$. In order to do that, we need to find the transmit and receive filters according to BD-ZF of all terminals for all possible SDMA groups and RBs;
2. For each RB, independently select the SDMA group with maximum total data rate, i.e., the SDMA group whose sum of the terminals' data rate is maximum.

Assuming that the estimated data rates $r_{g,j,n}$ are already calculated, i.e., the BD-ZF solution was already applied, the dominant operation of the optimal solution of URM problem is the computation of the total data rate of each SDMA group on each RB. According to the section 2.1, the maximum number of terminals in an SDMA group is limited by M_T. Therefore, the worst-case computational complexity of the optimal solution of the URM problem is $\mathcal{O}\left(GNM_T\right)$.

2.4. CRM problem and solution

According to [Lima et al. 2014], the CRM problem formulation is the same as the URM presented in (4) with the additional constraint that follows:

$$\sum_{j\in\mathcal{J}_s} u\left(\sum_{g\in\mathcal{G}}\sum_{n\in\mathcal{N}} x_{g,n}\, o_{g,j}\, r_{g,j,n}, t_j\right) \geq k_s,\ \forall s \in \mathcal{S}, \tag{5}$$

where $u(x, \beta)$ is a step function at β that assumes the value 1 if $x \geq \beta$ and 0 otherwise. Constraint (5) states that a minimum number of terminals should be satisfied for each service.

The optimal solution of CRM problem was described in [Lima et al. 2014] where it was formulated as ILP that can be optimally solved by BB-based algorithms [Nemhauser and Wosley 1999]. Note that, as in the URM solution, it is necessary to calculate $r_{g,j,n}$ to solve the CRM problem. Therefore, we need to find the transmit and receive filters according to BD-ZF of all terminals for all possible SDMA groups and RBs. Assuming that the estimated data rates $r_{g,j,n}$ are already calculated, the worst-case complexity of the optimal solution of the CRM problem is $\mathcal{O}\left(2^{GN}\right)$ [Lima et al. 2014].

3. SDMA Group Selection Framework

Our main idea is to reduce the number of SDMA groups used in the solutions. In other words, we intend to select $p\,G$ SDMA groups to be used to calculate the optimal solutions of URM and CRM problems where $0 < p < 1$.

Figure 2 shows a flowchart of the proposed framework. In step (1) we should identify all possible SDMA groups that can be composed. Then in step (2) we calculate the estimated Signal-to-Noise Ratio (SNR) of each link BS-terminal on all RBs as follows: $\bar{\gamma}_{j,n} = \dfrac{P.|\bar{h}_{j,n}|^2}{N_o}$, where P is the transmit power per spatial subchannel (fixed), N_o is the average noise power in the bandwidth of an RB and $\bar{h}_{j,n}$ consists in the mean of the elements of matrix $\mathbf{H}_{j,n}$. The average SNR is used in step (3) where we calculate a mean data rate, $\widehat{r}_{j,n}$, of terminal j on RB n. This mean data rate is obtained by the link adaptation lookup tables that map SNR regions to MCSs. The calculated data rate for each terminal j on each RB n is used in step (4) to calculate the SDMA grouping metric (see later). Then, in step (5) we select the $p\,G$ SDMA groups on each RB n with higher values for the considered metric. The remaining steps are present in the original solutions of URM and CRM problems in [Lima et al. 2014]. In step (6), the spatial filters are calculated for the selected SDMA groups in step (5). In step (7), the spatial filters are used to calculate the transmit data rate of each terminal j when belonging to the SDMA group g on RB n, $r_{g,j,n}$. Those data rates are input to the solutions of problems URM and CRM in step (8).

In this work we propose and evaluate different metrics to be used in step (4). Assume hereafter that $\widetilde{\mathcal{J}}_g$ consists in the set of terminals that belongs to the g^{th} SDMA group. The considered metric for the g^{th} SDMA group on RB n, $m_{g,n}$, is presented in Table 1.

The proposed metrics explore the mean data rates considering the SNR and number of terminals, which are variables that cause impact in the performance of group. The

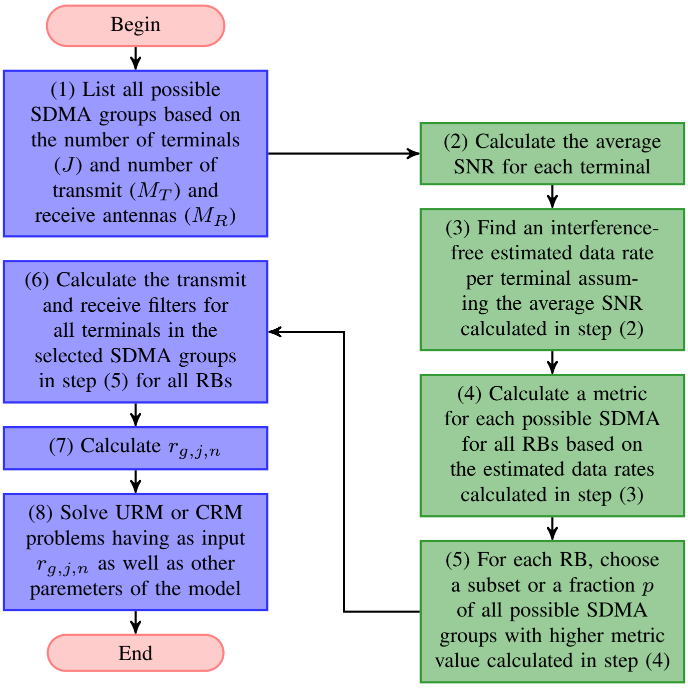

Figure 2. Flowchart of the proposed framework for solution of URM and CRM problems.

Table 1. Metrics for SDMA group selection.

Name	Equation	Comments
MEAN RATE	$m_{g,n} = \sum_{j\in\tilde{\mathcal{J}}_g} \widehat{r}_{j,n}$	Priority to SDMA groups with higher total transmit data rate
MAX MIN RATE	$m_{g,n} = \min_{j\in\tilde{\mathcal{J}}_g} (\widehat{r}_{j,n})$	Priority to SDMA groups with balanced terminal data rates or with a fair data rate distribution
SMALL GROUP MEAN RATE [1]	$m_{g,n} = \frac{\sum_{j\in\tilde{\mathcal{J}}_g} \widehat{r}_{j,n}}{\lvert\tilde{\mathcal{J}}_g\rvert}$	Variant of MEAN RATE that prioritizes the SDMA groups with small number of terminals
SMALL GROUP MAX MIN RATE	$m_{g,n} = \frac{\min_{j\in\tilde{\mathcal{J}}_g} (\widehat{r}_{j,n})}{\lvert\tilde{\mathcal{J}}_g\rvert}$	Variant of MAX MIN RATE that prioritizes the SDMA groups with small number of terminals

mean rate translate the channel quality. The number of terminal measures the spatial interference which degrades the channel quality.

For comparison purposes, we also consider a random scheme (RANDOM) where the choice of the $p\,G$ SDMA groups is done at random. With exception of the RANDOM strategy, the dominant operation in the computational complexity of the proposed selection metrics is the calculation of the average channel quality that requires $\mathcal{O}\left(M_R M_T\right)$. This operation is repeated for each terminal within the SDMA groups. As the number of terminals in an SDMA group is limited by M_T and the metric should be calculated for all SDMA groups on each RB, the worst-case computational complexity is $\mathcal{O}\left(G N M_T^2 M_R\right)$. Note that in some systems, an average wideband channel quality is already available and therefore, the complexity of this step is reduced to $\mathcal{O}\left(G N M_T\right)$.

4. Performance Results

[1] $|\cdot|$ represents the cardinality of a set.

Table 2. Main simulation parameters considered in the performance evaluation.

Parameter	Value	Unit
Cell radius	334	m
Transmit power per RB	0.8	W
Number of subcarriers per RBs	12	-
Number of RBs	10	-
Shadowing standard deviation	8	dB
Path loss [2]	$35.3 + 37.6 \cdot \log_{10}(d)$	dB
Noise spectral density	$3.16 \cdot 10^{-20}$	W/Hz
Number of snapshots	3000	-
Antenna configurations $M_R \times M_T$	2×2, 4×4 and 6×6	-
MIMO channel model	Classical IID	-
Number of services	2	-
Number of terminals per service	3	-
Required minimum number of satisfied terminals per service	2	- -

4.1. Simulation parameters

In this section we present the main assumptions that were considered for the performance evaluation of the proposed framework by means of computational simulations. We assume that the terminals are uniformly distributed in the cell coverage area. The links between BS and terminals are impaired by distance dependent path loss, lognormal shadowing and Independent and Identically Distributed (IID) Rayleigh-distributed fast fading. An RB consists of 12 subcarriers and 14 consecutive OFDM symbols [Lima et al. 2014]. We assume perfect Channel State Information (CSI) at the transmitter and receiver. We highlight here that CSI estimation is out of the scope of this work and the algorithms and methods proposed here do not rely on these aspects. Naturally, we expect that the absolute performance of the studied algorithms could be degraded when imperfect CSI estimation is present.

The simulations are organized by means of snapshots, where in a snapshot an RRA solution is provided. In every new snapshot, new independent samples of random variables are generated for channel state and other parameters of the model. The number of snapshots was chosen so as to assure statistical confidence of the presented results. We assume that the link adaptation is performed based on the report of 15 discrete Channel Quality Indicators (CQIs) used by the LTE system [Lima et al. 2014]. The Signal to Noise Ratios (SNRs) thresholds for MCS switching used here were the same ones employed in [Lima et al. 2014]. The main simulation parameters are shown in Table 2.

The ILP problems were solved using IBM ILOG CPLEX Optimizer [IBM 2009]. In the plots, we call "Original solution" the optimal solution of the CRM or URM problems that considers all possible SDMA groups. When the performance metrics are concerned, we consider two main ones: outage rate and total data rate. An outage event happens when an algorithm cannot manage to find a feasible solution for CRM problem, i.e., the algorithm does not find a solution fulfilling the constraints of CRM problem. Outage rate is defined as the ratio between the number of snapshots with outage events and the total number of simulated snapshots. Therefore, this performance metric shows the capability of the algorithms in finding a feasible solution to CRM problem. The outage

[2] d is the distance between the base station and the terminal in meters.

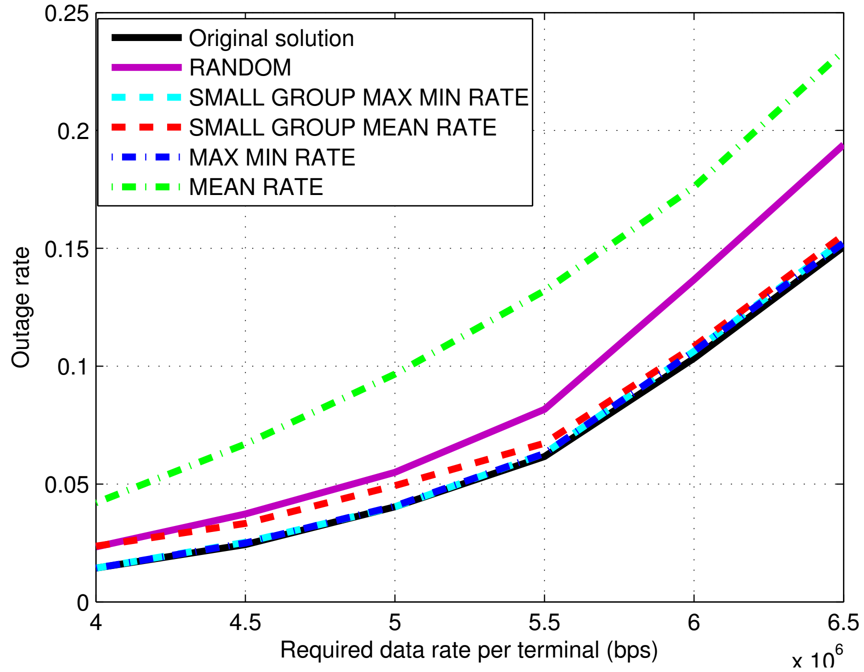

Figure 3. Outage rate versus required data rate with the optimal solution of the CRM problem with $p = 1$ (all SDMA groups are selected) and $p = 0.6$ for different metrics.

rate is meaningless for URM problem. The total data rate is the sum of the data rates obtained by all the terminals in the cell in a given snapshot. This metric is suitable for both URM and CRM problems. Finally, increments in the offered load are emulated by increasing the data rate requirements of the terminals.

4.2. Simulation Results

In Figures 3 and 4 we can see the outage rate versus the required data rate per terminal for the proposed framework with different proportion of selected groups: 60% and 30%, respectively. Let p the groups percentage considered in the problem. The objective is to compare the performance of the original solution of CRM problem where all SDMA groups are considered ($p = 1$), and the solutions using our proposed framework with different values of p with the metrics presented in section 3. As expected, the solution with $p = 1$ presents the best performance in terms of outage since it has more options of SDMA groups per RB in order to find an RRA solution that satisfies the problem constraints.

The worst performance in Figures 3 and 4 is achieved by the MEAN RATE metric. This metric selects the SDMA groups with higher mean data rate. Therefore, it is expected that most of the selected SDMA groups will be composed of the terminals with better channel qualities. Consequently, the RRA solution does not have many options to find feasible associations between RBs and SDMA groups that contain the other terminals in medium and poor channel states leading to a higher outage rate. The second worst performance is achieved with the RANDOM metric. Basically, this metric does not consider any information of terminals or channel states in order to select the SDMA groups.

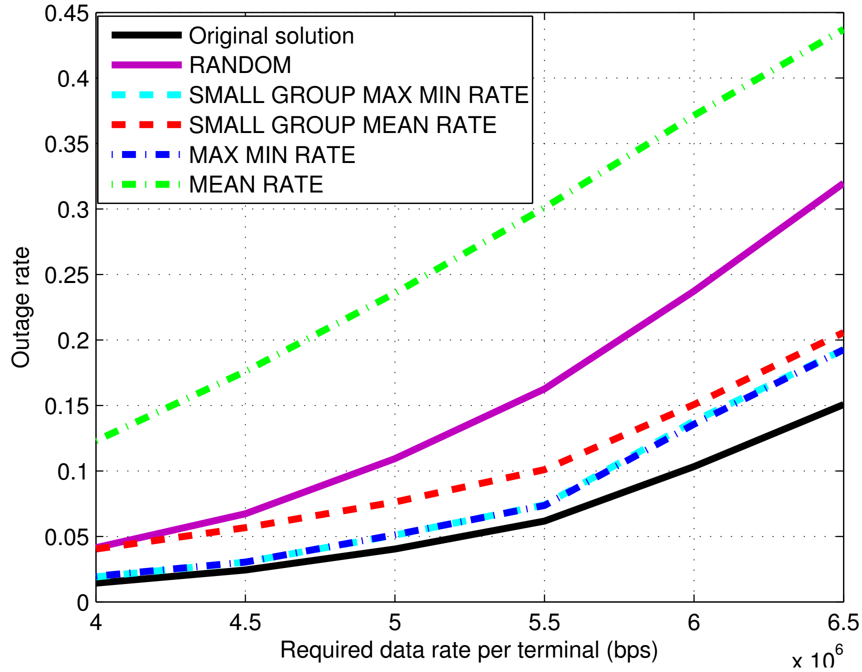

Figure 4. Outage rate versus required data rate with the optimal solution of the CRM problem with $p = 1$ (all SDMA groups are selected) and $p = 0.3$ for different metrics.

In both figures we can see that the SMALL GROUP MEAN RATE metric presents better performance than MEAN RATE metric. This metric SMALL GROUP MEAN RATE gives more opportunities to select terminals in medium and poor channel states since the data rate is not the unique factor to be considered. Focusing on Figure 3 we can see that the metrics MAX MIN RATE and SMALL GROUP MAX MIN RATE perform almost optimally. This means that the proposed framework achieves a practically optimal solution selecting only 60% of the SDMA groups. In Figure 4, we can observe again that the MAX MIN RATE and SMALL GROUP MAX MIN RATE present the best performance. By choosing the SDMA groups with maximum worst terminal data rate, those metrics are capable of providing a small and acceptable degradation in performance even when only 30% of the SDMA groups are considered. In particular, the SMALL GROUP MAX MIN RATE has the advantage of selecting SDMA groups with small size and therefore, the computational complexity of spatial filter calculation is reduced. It is worth of mentioning that all metrics present reduced performance loss compared to the solution with $p = 1$ as we increase the value of p. This can be seen by comparing Figures 3 and 4.

In Figure 5 we present the Cumulative Distribution Function (CDF) of the total data rate for the same scenario of Figure 4 when the required data rate per terminal is 6 Mbps. The samples used in the CDFs were taken from the snapshots in which all solutions managed to find a feasible solution, i.e., there is no outage. We can see that the solution with $p = 1$ presents the best performance, as expected. The poor performance of the RANDOM metric is due its unawareness regarding channel state information as explained before. All other metrics presents a similar performance with a loss in the 50^{th}

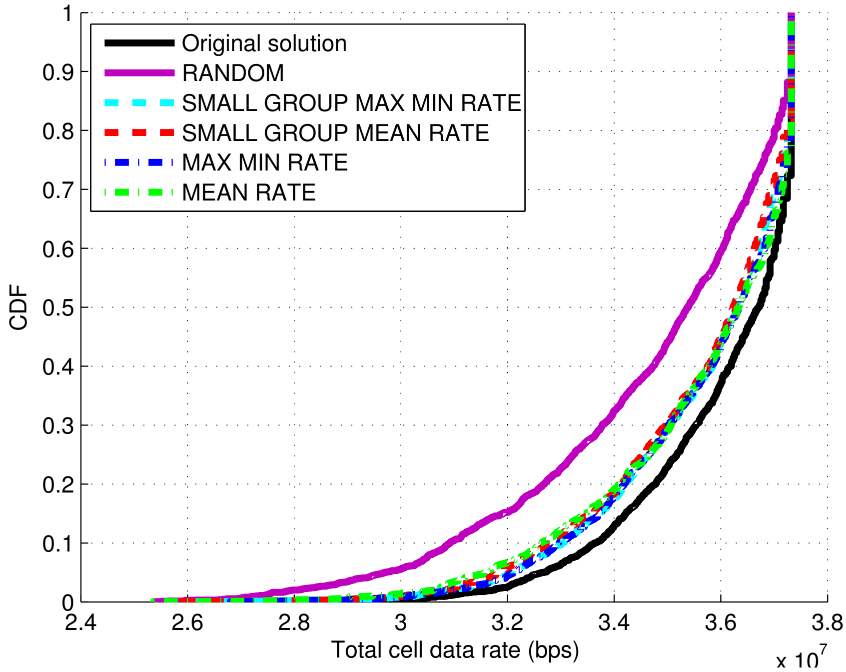

Figure 5. CDF of total data rate for the data rate requirement of 6 Mbps with the optimal solution of the CRM problem with $p = 1$ (all SDMA groups are selected) and $p = 0.3$ for different metrics.

percentile of only 1% compared to the original solution. Although it is not shown here, the performance loss of the proposed metrics are decreased as the terminals' required data rate is reduced.

In Figure 6 we present the mean of the total data rate for URM problem considering $p = 0.3$. As we can see in this figure, selecting only 30% of the SDMA groups for solution does not lead to significant performance losses compared to the original solution. Therefore, we will have an acceptable loss for a large reduction of groups. The exception is the RANDOM metric that presents a loss of 1.5 Mbps in the mean of the total data rate compared to the solution with $p = 1$, a reduction only 4%. The others metrics don't have loss greater than 0.7 Mbps. This performance is achieve because the problem don't have QoS restrictions. Users with good channel state will receive many RB, making easy increase the global rate.

In the following we provide some comments about the computational complexity of the studied solutions. When a fraction p of G possible groups is selected, we need to calculate the spatial filters of the terminals of only pG SDMA groups in the proposed framework for each RB. On the other hand, the original solution should calculate the spatial filters of the terminals of all G possible SDMA groups for each RB. According to the computational complexity of the BD-ZF presented in section 2.2, the proposed framework avoids the calculation of approximately $NG(1-p)M_R M_T^3 + NG(1-p)M_T^4$ operations compared to the original solution assuming that the maximum number of terminals in an SDMA group is M_T. Note that the complexity to calculate the metrics of all SDMA groups is a polynomial with order lower than the calculation of all the spatial filters with

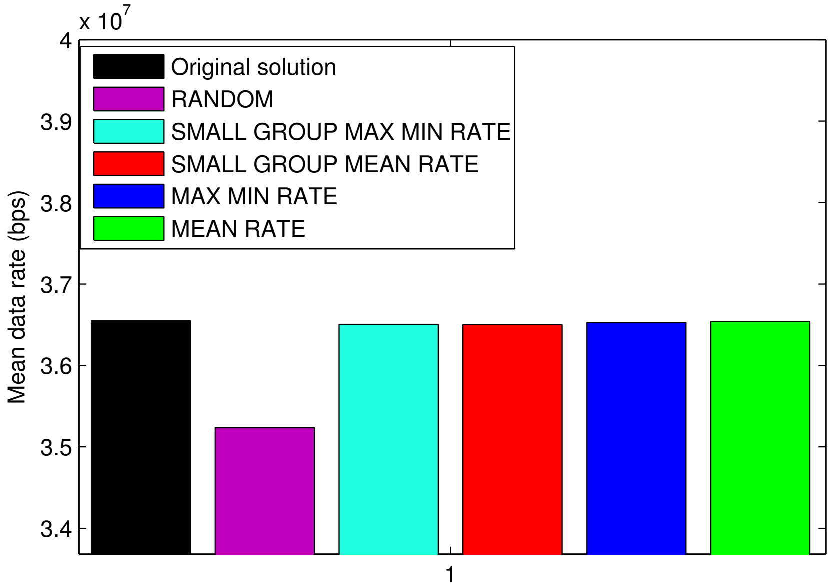

Figure 6. MEAN of total data rate with the optimal solution of the URM problem with $p = 1$ (all SDMA groups are selected) and $p = 0.3$ for different metrics.

BD-ZF as shown in section 3.

Besides, the reduction in computational complexity is also present in the optimal CRM and URM solutions. As it was shown in section 2.4, the complexity of the original solution of CRM problem is $\mathcal{O}\left(2^{GN}\right)$ and with the proposed framework is $\mathcal{O}\left(2^{pGN}\right)$. The worst-case complexity of the original solution of URM problem is $\mathcal{O}\left(GNM_T\right)$ and with proposed framework is $\mathcal{O}\left(pGNM_T\right)$.

The presented results in this section have shown that significant computational work can be saved by selecting only a fraction of the possible SDMA groups when solving the CRM and URM problems. This selection should be done with the help of some of the metrics proposed in this work. In particular, the SMALL GROUP MAX MIN RATE has shown to be effective for both problems.

5. Conclusions and Perspectives

Although Radio Resource Allocation (RRA) algorithms applied in Multiple Input Multiple Output (MIMO) systems can lead to important performance gains, they are in general very complex and impose a high computational burden. Therefore, strategies that decrease the computational complexity with low performance loss are welcome. In this work we revisited the Constrained Rate Maximization (CRM) and Unconstrained Rate Maximization (URM) problems. In order to obtain the optimal solution of those problems, it is needed to list all possible Space Division Multiple Access (SDMA) groups and calculate the transmit and receive filters of the terminals. Our proposed was to select only a fraction of all the possible SDMA groups based on specific metrics based on Channel State Information (CSI) in order to solve CRM and URM problems. Accordingly, there is an overall computational complexity reduction compared to the original solution.

By computer simulations, we have shown that some of the proposed selection metrics are capable of providing a small performance degradation compared to the original solution even when only 30% of the possible SDMA groups are selected. Therefore, the proposed framework is capable of achieving a good performance-computational complexity trade off. As the proposed framework is a flexible and general, we point out as perspectives the study of the proposed framework on other RRA problems for MIMO systems.

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